

Knots

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Abstract

These are lecture notes of a course on knot theory given at Cardiff University in the autumn term 2015/16. The content is mostly based on lectures by D. Evans. Many of the pictures have been created with `knotplot`.

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