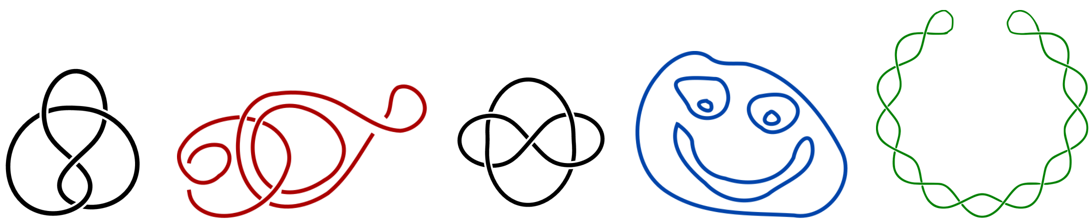
Knots – Exercise Sheet 4

Module MA0322 – Gandalf Lechner – Autumn 2015

handed out: 10 November
solutions can be handed in until: 19 November
will be discussed in class: 20 November

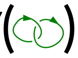
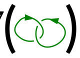
Problem 11: Jones and HOMFLY Polynomials

Compute the Jones and HOMFLY polynomials of the following five links (you need to pick an orientation)



Problem 12: Conway and Alexander polynomials

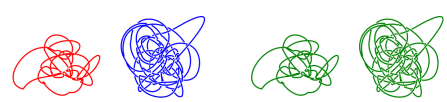
Compute the Conway polynomials $\nabla_{3_1}(z), \nabla_{4_1}(z)$ of the (oriented) trefoil 3_1 and figure of eight knots 4_1 (choose orientations). You may use the Conway polynomials of the Hopf links,

$\nabla\big($  $\big) = z$ $\nabla\big($  $\big) = -z$

Then replace the variable z by $t^{-1/2} - t^{1/2}$ and compare with the Alexander polynomials of the trefoil (computed in lecture 8) and the figure of eight (computed in problem 6).

Problem 13: The Jones polynomial and composite knots

Show that for any two oriented knots K, K', there holds the equation $V_{K\#K'}(t) = V_K(t) \cdot V_{K'}(t)$ between the three Jones polynomials $V_{K\#K'}, V_K, V_{K'}$.



To do so, you can proceed as follows:

a) Use the skein relation of the Jones polynomial to express $V_{K\#K'}$ in terms of the Jones polynomial $V_{K\sqcup K'}$ of the 2-link $K \sqcup K'$, which consists of the two components K, K', with no crossings between K and K' (see the schematic picture above).

b) Consider the bracket polynomial, and prove that $\langle K \sqcup K' \rangle = d \, \langle K \rangle \cdot \langle K' \rangle$ as polynomials in A, B, d, by working with sums over states. Deduce an analogous equation for the Kauffman bracket.

c) Show that the writhe satisfies $w(K \sqcup K') = w(K) + w(K')$, and use this and part b) to find a relation expressing $V_{K\sqcup K'}$ in terms of V_K and $V_{K'}$.

d) Use a) and c) to show the claimed equation $V_{K\#K'} = V_K \cdot V_{K'}$.

You can also test the validity of $V_{K\#K'} = V_K \cdot V_{K'}$ by considering examples for K, K'.