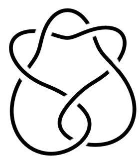
Knots – Exercise Sheet 3

Module MA0322 – Gandalf Lechner – Autumn 2015

handed out: 27 October
solutions can be handed in until: 5 November
will be discussed in class: 6 November

Problem 8: Colouring groups

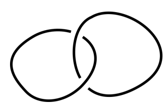
Compute the colouring group of the knot K with the diagram



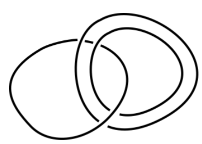
That is, find integers $d_1, ..., d_n \in \mathbb{N}_0$ such that $\mathrm{Col}(K) \cong \mathbb{Z}/d_1\mathbb{Z} + ... + \mathbb{Z}/d_n\mathbb{Z}$.

Problem 9: Bracket polynomials

a) Compute the bracket polynomial $\langle D \rangle$ as a polynomial in A, B, d, from its definition as a sum over states, for the standard diagram D of the Hopf link, namely

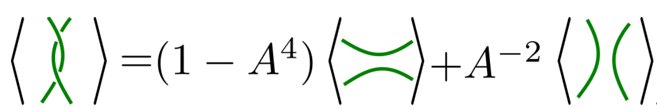


b) Compute the bracket polynomial $\langle D \rangle$ as a polynomial in A, B, d by using its recursion relation for the diagram

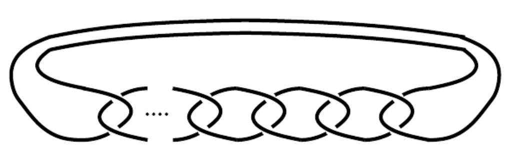


Problem 10: Brackets and bracelets

a) Prove that the following relation holds for the Kauffman bracket (i.e., the bracket polynomial with $B = A^{-1}$ and $d = -A^2 - A^{-2}$)



b) ⋆ Calculate the Kauffman bracket of the "bracelet" link diagram D_n with $n \in \mathbb{N}$ components, depicted below.



Knots – Quiz 2

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Here are some quick quiz questions to test your understanding of the material presented in the second chapter. These questions are voluntary, and will not be handed in or marked. The quiz is meant as a (highly recommended) test for yourself.

1) Define what it means for a knot to be 3-colourable. Give an example of a knot that is 3-colourable, and an example of a knot that is not 3-colourable.

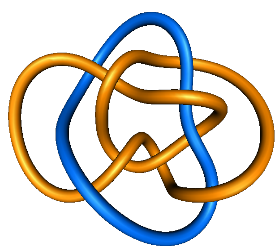
2) Define the determinant of a link.

3) Let K be a knot with determinant 195. For which integers p is K p-colourable?

4) Does there exist a link that is 7-colourable, but not 21-colourable? Why?

5) Show that the property of 3-colourability does not change under Reidemeister moves of type II.

6) A friend has been working on a puzzle for two weeks without solving it. The puzzle consists of two interlocked strings:



The task is to disentangle the blue string from the yellow string, i.e. to “free” the blue part (without cutting the yellow string, of course). Having heard that you know about knot theory, your friend comes to you and asks for your help. How would you help him?

7) If $\mathrm{Col}(L) \cong \mathbb{Z}/3\mathbb{Z} + \mathbb{Z}/3\mathbb{Z} + \mathbb{Z}/15\mathbb{Z}$, for which primes p is L p-colourable?

8) Let K be a knot with crossing number $c(K) = 5$ and determinant $\det K = 105$. What are the possible forms of the colouring group $\mathbb{Z}/e_1\mathbb{Z} + ... + \mathbb{Z}/e_n\mathbb{Z}$ of K?

9) Explain why the Alexander polynomial is a stronger invariant than the determinant.

10) Call a knot K *invertible* if it has the property that once given an orientation, it is ambient isotopic to itself, but with the opposite orientation. Can $t^3 + t + t^{-1}$ be the Alexander polynomial of an invertible knot?