Knots – Exercise Sheet 2

Module MA0322 – Gandalf Lechner – Autumn 2015

handed out: 13 October
solutions can be handed in until: 22 October
will be discussed in class: 23 October

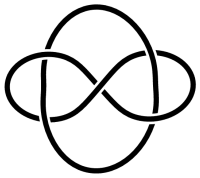
Problem 4: Invariance properties of writhe

Give a proof of Lemma 1.16 by using Reidemeister moves.

Lemma 1.16: *Writhe is an invariant of oriented links under regular isotopy, but not under ambient isotopy.*

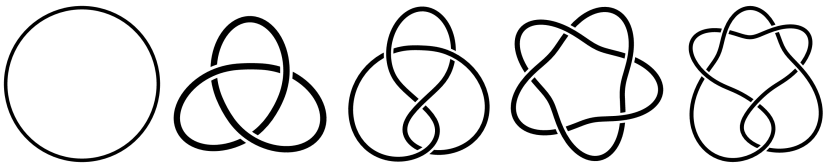
Problem 5: Non-triviality of the Whitehead link

Use colourability properties to show that the Whitehead link (depicted on the right) is non-trivial, i.e., not equivalent to the 2-unlink.



Problem 6: Colourability, Determinants, and the Alexander Polynomial

Consider the following five knot diagrams



0_1 3_1 4_1 5_1 6_1

a) Working with colourings, show that 0_1, 4_1 and 5_1 are not 3-colourable, but 3_1 and 6_1 are.

b) Working with determinants, show that no two of the shown diagrams belong to equivalent knots except maybe the pair 4_1, 5_1.

c) Working with the Alexander polynomial, show that also the knots represented by the diagrams 4_1 and 5_1 are not equivalent.

Problem 7⋆: The Alexander Polynomial and Mirror Images

Show that given an oriented topological knot $\mathcal{K}$ and its mirror image $\mathcal{K}^*$ (defined in problem 3 on sheet 1), there exist diagrams, D of $\mathcal{K}$, D^* of $\mathcal{K}^*$, and labelings of (the crossings and arcs of) D, D^* such that the Alexander polynomials Δ_D and Δ_{D^*} coincide. Thus the Alexander polynomial can not distinguish between a knot and its mirror image.

Hint: Use the freedom of choosing the reflection plane to find a simple connection between the matrices underlying Δ_D and Δ_{D^*}.

Knots – Quiz 1

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Here are some quick quiz questions to test your understanding of the material presented in the first chapter. These questions are voluntary, and will not be handed in or marked. The quiz is meant as a (highly recommended) test for yourself.

1) Explain the difference between a geometrical and topological knot. Is any topological knot also a geometrical one?

2) Explain the difference between a knot and a link. Is every link also a knot?

3) Give an example of a link invariant.

4) Draw diagrams of the unknot, the trefoil, the Whitehead link, the Hopf link, the 2-unlink, and the figure eight knot.

5) How does the writhe of an oriented knot change when the orientation of the knot is reversed?

6) State Reidemeister's Theorem.

7) Given two diagrams, how would you proceed to show that these diagrams are equivalent?

8) Given two diagrams, how would you proceed to show that these diagrams are not equivalent?

9) Explain the differences between planar, regular, and ambient isotopy of diagrams. Can one deduce some of these equivalences from others in the list? (For example, are two diagrams that are regular isotopic also automatically ambient or planar isotopic?)

10) Show that the map I from the set of all geometrical links to the set of all topological links, which maps each link L to its equivalence class $[L]$, i.e. $I(L) := [L]$, is a complete link invariant. Discuss the usefulness of this invariant.