Knots – Exercise Sheet 1

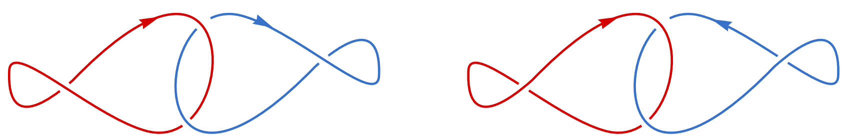
Module MA0322 – Gandalf Lechner – Autumn 2015

Problem 1: Crossing Number

Show that there exists no geometrical knot with crossing number two. What if you allow for geometrical links?

Problem 2: Writhe and Linking Number

Compute the writhe and linking number of both the following oriented link diagrams. Are these diagrams equivalent?



Problem 3: Mirror Images of Links and Chirality

For a two-dimensional plane E in $\mathbb{R}^3$, we denote by $R_E : \mathbb{R}^3 \to \mathbb{R}^3$ the reflection about E.

a) Given a geometrical link L and any two planes $E, E' \subset \mathbb{R}^3$, show that the reflected links $R_E(L)$ and $R_{E'}(L)$ are equivalent.
Hint: This requires a bit of linear algebra/geometry. What kind of transformation is $R_E \cdot R_{E'}$?

b) We define the mirror image $\mathcal{L}^*$ of a *topological* link $\mathcal{L}$ as follows. If $\mathcal{L} = [L]$ for some geometrical link L, we set $\mathcal{L}^* := \{L' : L' \cong R_E(L)\} = \{\text{all links equivalent to } R_E(L)\}$, where E is some two-dimensional plane in $\mathbb{R}^3$. Show that this definition does not depend on the choice of the plane E, and also not on the choice of the representative L within the class $\mathcal{L}$.

c) Show that the diagrams of $\mathcal{L}^*$ are precisely the diagrams of $\mathcal{L}$, but with all overcrossings replaced by undercrossings and vice versa.

d) A link $\mathcal{L}$ is called *chiral* if $\mathcal{L} \neq \mathcal{L}^*$ (such links occur in different "left" and "right" versions) and *achiral* (or *amphichiral*) if $\mathcal{L} = \mathcal{L}^*$. Show that the figure eight knot is achiral by working with a model of string/cable – memorise the necessary moves so that you can demonstrate achirality. Be sure not to knot a trefoil instead of a figure eight or you will have a hard time!