Borchers triples and the construction of models in quantum field theory

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UNIVERSITÄT LEIPZIG

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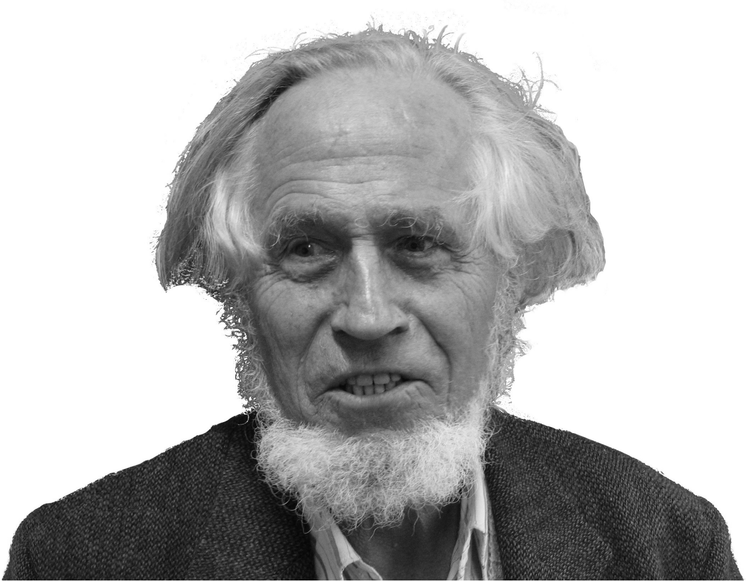

Hans-Jürgen Borchers (2005)

Outline of the talk

1) (Algebraic) QFT and the construction of models
2) Borchers triples and Borchers' theorem
3) Several examples of Borchers triples and concrete construction procedures

Construction of QFTs by quantization

- Traditional construction of QFTs: Start from a classical Lagrangian field theory, obtain the QFT by quantization and perturbative renormalization
  - Most successful in low dimensions ($d = 1 + 1$ or $d = 1 + 2$), where complete constructions have been achieved and perturbation expansion could be "tamed"

    [Glimm/Jaffe, and many more, since late 1960s]
  - State of the art in $d = 1 + 3$: Rigorous construction of many interacting theories in a formal power series setting

    [talk by Kopper, Brunetti/Dütsch/Fredenhagen, ...]

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  - State of the art in $d = 1+3$: Rigorous construction of many interacting theories in a formal power series setting

    [talk by Kopper, Brunetti/Dütsch/Fredenhagen, ...]

Here:

- Discussion of alternative, operator-algebraic construction procedures
- Long term aim: Better understand the quantum structure of local interactions

Algebraic description of QFT on Minkowski space

Setting

- Hilbert space $\mathcal{H}$ (physical states given by vectors and density matrices in $\mathcal{H}$)
- Unitary representation $(x, \Lambda) \mapsto U(x, \Lambda)$ of the Poincaré group on $\mathcal{H}$ (relativistic symmetry)
- Require generator of time translations to have positive spectrum in each Lorentz frame (Spectrum condition, vacuum)
- Vacuum vector $\Omega \in \mathcal{H}$, invariant under U
- For each region $O \subset \mathbb{R}^d$ in Minkowski space $\mathbb{R}^d$, a von Neumann algebra $\mathcal{A}(O) \subset \mathcal{B}(\mathcal{H})$ of observables measurable in O (can, but need not be generated by quantum fields)
- The observable net $O \mapsto \mathcal{A}(O$ has to satisfy some physical requirements (in particular, Poincaré covariance and Einstein locality)

Algebraic description of QFT on Minkowski space

- For computing physically interesting quantities such as for example cross sections, these data are sufficient [Haag]
  -- no quantum or classical fields necessary
- For general analysis, the "more invariant" algebraic point of view has proven advantageous (e.g. Borchers classes)

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Main open problem:

$(\mathcal{H}, U, \Omega, O \mapsto \mathcal{A}(O))$ is a very complex structure
... rigorous construction of examples is a challenge.

A seminal paper of Borchers

Commun. Math. Phys. 143, 315–332 (1992)

Communications in Mathematical Physics

The CPT-Theorem in Two-dimensional Theories of Local Observables

H. J. Borchers

Institut für Theoretische Physik, Universität Göttingen, Bunsenstrasse 9, W-3400 Göttingen, Federal Republic of Germany

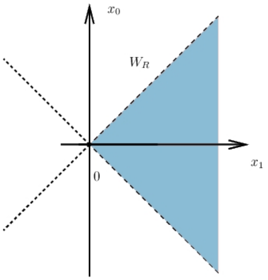
Received January 23, 1991

Abstract. Let $\mathcal{M}$ be a von Neumann algebra with cyclic and separating vector Ω, and let $U(a)$ be a continuous unitary representation of $\mathbf{R}$ with positive generator and Ω as fixed point. If these unitaries induce for positive arguments endomorphisms of $\mathcal{M}$ then the modular group act as dilatations on the group of

Wedges

Certain special regions in Minkowski space (wedges) play a distinguished role here.

$$W_0 := \{x \in \mathbb{R}^d \,:\, x_1 > |x_0|\} \qquad \text{"standard wedge"}$$



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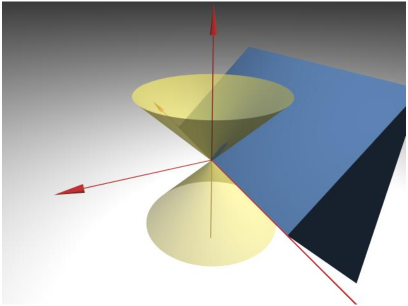

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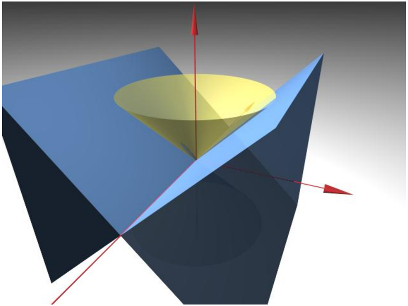

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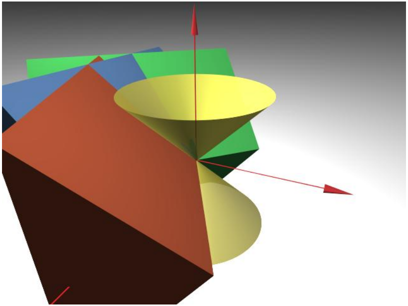

Borchers triples

Definition

A Borchers triple $(\mathcal{M}, U, \Omega)$ consists of

- A von Neumann algebra $\mathcal{M} \subset \mathcal{B}(\mathcal{H})$ on some Hilbert space $\mathcal{H}$,
- A strongly continuous unitary representation of the translations $\mathbb{R}^d$ on $\mathcal{H}$,
  - with spectrum condition,
  - such that $U(x)\mathcal{M}U(x)^{-1} \subset \mathcal{M}$ for $x \in W_0$,
- A unit vector $\Omega \in \mathcal{H}$ which is invariant under U and cyclic and separating for $\mathcal{M}$.

- Name "Borchers triple" introduced by Buchholz [2010]
- Structure of Borchers triples much easier than that of full-fledged QFTs (single algebra $\mathcal{M}$ instead of infinite collection $\{\mathcal{A}(O\}_O$)

"Borchers' Theorem"

Theorem [Borchers 1992]

Let $(\mathcal{M}, U, \Omega)$ be a Borchers triple and consider the modular group Δ^{it} and modular conjugation J of the pair $(\mathcal{M}, \Omega)$. Then

$$\Delta^{it} U(x) \Delta^{-it} = U(\Lambda_1(t)x),$$
$$JU(x)J = U(j_1 x),$$

where $\Lambda_1(t)$ is the Lorentz boost in x_1-direction with rapidity $-2\pi t$, and j_1 the reflection at the edge of W_0.

- Generalizes an essential part of the Bisognano-Wichmann theorem to algebraic QFT
- Can be used, with further assumptions, to prove modular covariance, spin-statistics, CPT theorems [in particular Mund 2001]
- **Focus here:** Use of Borchers triples / Borchers' theorem in constructive approaches.

About the proof

- Borchers found his original proof using analytic functions in several complex variables, one of his areas of expertise and favorite tools

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We learn from these results that one can interprete $U(\Lambda(t))$ as the modular group of the wedge algebra and that J can be interpreted as its modular conjugation.

For the modular group of the forward lightcone or the double cone in conformal field theory consult the original papers ([Bu], [HL]).

D. *Remarks on the edge of the wedge problem*

The theory of several complex variables is an important tool in quantum field theory and we assume familiarity with these methods. The situation appearing here (and often in other physical problems) is the edge of the wedge problem. One deals with two analytic functions $f^+(z)$ and $f^-(z)$, $z \in \mathbb{C}^n$ defined in tubes T^+ and $T^- = -T^+$ respectively. The tube T^+ is based on a convex cone $C \subset \mathbb{R}^n$ with apex at the origin and defined by:

$$T^+ = \{z \in \mathbb{C}^n;\ z = x + iy,\ y \in C,\ x \in \mathbb{R}^n\}.$$

One assumes that $f^+(z)$ and $f^-(z)$ both have boundary values $f^+(x)$, $f^-(x)$ respectively (in the sense of distributions) and that these boundary

[Borchers, Ann. Poincare Phys. Theor. 63, 1995, 331-382]

- Later a simpler alternative proof was found by Florig [Florig 1998]

From Borchers triples to local nets

In $d = 1 + 1$ dimensions, a Borchers triple $(\mathcal{M}, U, \Omega)$ gives rise to an associated local net $\mathcal{A}$ (QFT model):

- $\mathcal{A}(W_0) := \mathcal{M}$
- $\mathcal{A}(W_0 + x) := U(x)\mathcal{M}U(x)^{-1}$
- $\mathcal{A}(-W_0 + x) := U(x)\mathcal{M}'U(x)^{-1}$
- $\mathcal{A}((W_0 + x) \cap (-W_0 + y)) := (U(x)\mathcal{M}U(x)^{-1}) \cap (U(y)\mathcal{M}'U(y)^{-1})$

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- By Borchers' theorem, the so defined net $O \mapsto \mathcal{A}(O)$ transforms covariantly under a representation of the Poincaré group built from U and the modular data of $(\mathcal{M}, \Omega)$.
- Moreover, $\mathcal{A}$ satisfies Einstein locality.
- Non-triviality of the intersections $\mathcal{A}((W_0 + x) \cap (-W_0 + y))$ is however not automatic here – additional work necessary [Doplicher/Longo 1984, Buchholz/D'Antoni/Longo 1990 → Buchholz/GL 2004].

Borchers triples in higher dimensions

In $d > 1+1$ dimensions, the geometric structure of the family of wedges is different - need adapted definition of Borchers triple
[Baumgärtel/Wollenberg 1992, Buchholz/Summers 2008]

Definition

A causal Borchers triple $(\mathcal{M}, U, \Omega)$ consists of

- A von Neumann algebra $\mathcal{M} \subset \mathcal{B}(\mathcal{H})$ on some Hilbert space $\mathcal{H}$,
- A strongly continuous unitary representation of the Poincaré group on $\mathcal{H}$,
  - with spectrum condition,
  - such that $U(x,\Lambda)\mathcal{M}U(x,\Lambda)^{-1} \subset \mathcal{M}$ whenever $\Lambda W_0 + x \subset W_0$,
  - such that $U(x,\Lambda)\mathcal{M}U(x,\Lambda)^{-1} \subset \mathcal{M}'$ whenever $\Lambda W_0 + x \subset -W_0$,
- A unit vector $\Omega \in \mathcal{H}$ which is invariant under U and cyclic and separating for $\mathcal{M}$.

- Constructions of nets from a causal Borchers triple works analogously to the two-dim. case.

Constructive algebraic QFT

Possible construction strategy for QFT models:

1. Construct a Borchers triple based on physical input
2. Analyze the local observable content of the associated net

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- Borchers triples more manageable than full QFTs
- Many examples exist
- First general structure results [Longo/Witten 2010]
- Efficient tools for analyzing local observable content currently restricted to $d = 1 + 1$

Constructive algebraic QFT

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» Rest of the talk: Examples of Borchers triples and associated QFT models (1.)

A purely quantum construction of free theories

[Schroer 1997], [Brunetti/Guido/Longo 2002]

- Input: Hilbert space $\mathcal{H}_1$ with unitary positive energy representation U_1 of the proper Poincaré group. *(fixes particle content)*
  - Boosts in x_1-direction: $\Lambda_1(t)$, reflection j_1 at edge of W_R.
- Define

$$\Delta^{it} := U_1(0, -2\pi t)\,, \qquad J := U_1(j_1)\,, \qquad S := J\Delta^{1/2}\,.$$

- S is densely defined, closable, antilinear involution
- Consider real subspace

$$\mathcal{K} := \{\psi \in \text{dom}\, S \,:\, S\psi = \psi\} \subset \mathcal{H}_1$$

"one-particle vectors localized in W_0"

A purely quantum construction of free theories

The pair $(\mathcal{K}, U_1)$ is a single particle version of a Borchers triple:

- $U_1(x, \Lambda)\mathcal{K} \subset \mathcal{K}$ whenever $\Lambda W_0 + x \subset W_0$.
- $U_1(x, \Lambda)\mathcal{K} \subset \mathcal{K}'$ (sympl. compl.) whenever $\Lambda W_0 + x \subset -W_0$.
- $\mathcal{K}$ is "standard", i.e. $\overline{\mathcal{K} + i\mathcal{K}} = \mathcal{H}_1$ and $\mathcal{K} \cap i\mathcal{K} = \{0\}$.

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Theorem [Brunetti/Guido/Longo 2002]

Let $\mathcal{H} = \Gamma(\mathcal{H}_1)$ denote the Fock space over $\mathcal{H}_1$, with Fock vac. Ω, second quantized representation $\Gamma(U_1)$, and

$$\mathcal{M}_0 := \{V(\psi) \,:\, \psi \in \mathcal{K}\}'' \qquad (V(\psi) \text{ Weyl operator})\,.$$

Then

- $(\mathcal{M}_0, \Gamma(U_1), \Omega)$ is a Borchers triple on $\mathcal{H}$.
- In the associated net $\mathcal{A}$, the vacuum is cyclic for $\mathcal{A}(C)$ for each spacelike cone C.

- This procedure yields any free field theory by a purely quantum construction.
- For interacting theories, some additional input is necessary.

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U_1		$\mathcal{K} \subset \mathcal{H}_1$		$(\mathcal{M}_0, \Gamma(U_1), \Omega)$
single particle spectrum	$\Rightarrow$	one-particle local. structure	$\Rightarrow$	“free” Borchers triple, free QFT

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single particle spectrum	$\Rightarrow$	one-particle local. structure	$\Rightarrow$	"free" Borchers triple, free QFT

$\Downarrow$

$(\mathcal{M}, U, \Omega)$
"interacting" Borchers triple and QFT

- Study deformations of Borchers triples, in general or in particular for $(\mathcal{M}_0, \Gamma(U_1), \Omega)$

A general deformation procedure

[Buchholz/GL/Summers 2011]

Input:

- A Borchers triple $(\mathcal{M}, U, \Omega)$ in any dimension $d \geq 1+1$
- A skew-symmetric real $(d \times d)$-matrix Q as deformation parameter

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Definition:

The warped convolution of a (smooth) operator $A \in \mathcal{B}(\mathcal{H})$ is

$$A_Q := (2\pi)^{-d} \iint dp\, dx\, e^{-ipx}\, U(Qp, 1) A U(Qp, 1)^{-1} U(x, 1)$$

- A_Q can be defined in an oscillatory sense on smooth vectors
- Idea for this deformation comes from QFT on noncommutative spacetimes [Grosse/GL 2007], transferred to operator-algebraic setting [Buchholz/Summers 2008]

A general deformation procedure

Properties of the deformation $A \mapsto A_Q$:

- $A_0 = A$
- $A \mapsto A_Q$ is linear
- ${A_Q}^* = {A^*}_Q$
- $A_Q B_Q = (A \times_Q B)_Q$ with

$$A \times_Q B = (2\pi)^{-d} \int dp\, dx\, e^{-ipx}\, U(Qp)AU(Qp)^{-1}\, U(x)BU(x)^{-1}$$

Rieffel product [Rieffel 1992]

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  Rieffel product [Rieffel 1992]

- $A_Q \Omega = A\Omega$
- $U(x,\Lambda)A_Q U(x,\Lambda)^{-1} = \big(U(x,\Lambda)AU(x,\Lambda)^{-1}\big)_{\Lambda Q \Lambda^{-1}}$

A general deformation procedure

- **Observation 1:**The matrix Q can be chosen in such a way that

$$\Lambda Q \Lambda^{-1} = Q \Leftrightarrow \Lambda W_0 = W_0 \,,$$
$$\Lambda Q \Lambda^{-1} = -Q \Leftrightarrow \Lambda W_0 = -W_0 \,.$$

[Grosse/Lechner 2007]

- **Observation 2:** If $QV_+ \subset W_0$, by exploiting the spectrum condition one can show

$$[A_Q, B'_{-Q}] = 0 \qquad \text{for } A \in \mathcal{M},\ B' \in \mathcal{M}' \,.$$

[Buchholz/Summers 2008]

A general deformation procedure

Theorem [Buchholz/GL/Summers 2011]

Let $(\mathcal{M}, U, \Omega)$ be a Borchers triple, and Q appropriately chosen. Then, with

$$\mathcal{M}_Q := \{A_Q \,:\, A \in \mathcal{M} \text{ smooth}\}'' ,$$

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also $(\mathcal{M}_Q, U, \Omega)$ is a Borchers triple.

- Warped convolution produces new (non-equivalent) examples of Borchers triples from known ones.
- Effect of deformation on scattering states: Two-particle S-matrix changes (picks up an energy-dependent phase)
- Effect of deformation on thermodynamics: Under investigation [work in progress with Schlemmer]
- Drawback of this construction: In $d > 1+1$, the net associated with the deformed triple probably has no strictly local observables

A deformation of the massive free Borchers triple

Now another example of a deformation, but making use of the more particular "second quantized" structure of $(\mathcal{M}_0, \Gamma(U_1), \Omega)$.

Definition

- A symmetric inner function [Longo/Witten 2010, Longo/Rehren 2011] is a bounded analytic function φ on the upper half plane such that $\overline{\varphi(t)} = \varphi(t)^{-1} = \varphi(-t)$ for $t \in \mathbb{R}$.
- A root of a symmetric inner function is a function $R \in L^\infty(\mathbb{R})$ such that $\overline{R(t)} = R(t)^{-1} = R(-t)$ and R^2 is symmetric inner.

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- Properties of R closely related to S-matrix properties (unitarity, crossing symmetry..)
- Analyticity of R^2 necessary for locality (moving wedge localization in commutators apart, half-sided Fourier transform)

A deformation of the massive free Borchers triple

Choose a root R, a matrix Q as before, and define on Fock space $\Gamma(\mathcal{H}_1)$ over $\mathcal{H}_1 = L^2(\mathbb{R}^{d-1}, d\mu_m(p))$ the operators

$$[T_R(q)\Psi]_n(p_1...p_n) := \prod_{k=1}^{n} R(qQp_k) \cdot \Psi_n(p_1...p_n).$$

Deform free field operators $\phi(\psi) = a^\dagger(\psi) + a(\psi)$ according to

$$\phi_R(\psi) := a_R^\dagger(\psi) + a_R(\psi)\,,$$

$$a_R(p) := a(p)\,T_R(p)\,, \qquad a_R(\psi) = \int d\mu_m(p)\,a_R(p)\overline{\psi(p)}$$

and similarly for field polynomials (Borchers-Uhlmann algebra)

A deformation of the massive free Borchers triple

Theorem [GL 2011], generalizations by [Alazzawi 2012]

Let R be a root. With

$$\mathcal{M}_R := \{e^{i\phi_R(\psi)} \,:\, \psi \in \mathcal{K}\}'' ,$$

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- Two-particle S-matrix essentially given by R^2.
- Structural properties of the associated net similar to warping case.
- In $d = 1 + 1$, further properties known for a large family of R's: Existence of local observables, asymptotic completeness, solution of inverse scattering problem for factorizing S-matrices, scaling limits, expansions of local operators ...
  [Schroer 1997, Borchers/Buchholz/Schroer 2000, GL 03, Buchholz/GL 04, GL 06, Bostelmann/GL/Morsella 2011, GL/Schützenhofer 2012, Bostelmann/Cadamuro 2012,...]

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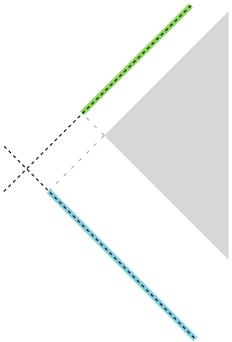
- **Conjecture:** Any integrable QFT can be obtained like this.

The chiral situation

» In the massless, two-dimensional situation, special chiral examples exist (decomposition in left- and right-movers). For a Borchers triple $(\mathcal{M}, U, \Omega)$, this means

- $\mathcal{H} \cong \mathcal{H}_+ \otimes \mathcal{H}_-$
- $\mathcal{M} \cong \mathcal{M}_+ \otimes \mathcal{M}_-$
- $U(x) \cong U_+(x_-) \otimes U_-(x_+)$,
  $x_\pm := x_0 \pm x_1$
- $\Omega \cong \Omega_+ \otimes \Omega_-$



Deformations of chiral Borchers triples

Theorem [Tanimoto 2011]

Let φ be a symmetric inner function, $(\mathcal{M}_+ \otimes \mathcal{M}_-, U_+ \otimes U_-, \Omega_+ \otimes \Omega_-)$ the chiral Borchers triple of the free massless field in two dimensions, and

$$[S_\varphi \Psi]_{n,m}(p_1 \dots p_n; q_1 \dots q_m) := \prod_{\substack{i=1\dots n \\ j=1\dots m}} \varphi_0(p_i, q_j) \cdot \Psi_{n,m}(p_1 \dots p_n; q_1 \dots q_m).$$

Then

$$((\mathcal{M}_+ \otimes 1) \vee S_\varphi(1 \otimes \mathcal{M}_-)S_\varphi^*, U_+ \otimes U_-, \Omega_+ \otimes \Omega_-)$$

is a Borchers triple, with wave-S-matrix S_φ [Buchholz 1975, Dybalski/Tanimoto 2010].

- Uses Longo-Witten endomorphisms [Longo/Witten 2010].
- Further constructions by [Dybalski/Tanimoto 10, Bischoff/Tanimoto 11/12].

Equivalence of two deformations

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Theorem [GL/Schlemmer/Tanimoto 2012] – arXiv:1209.2547

Let φ be a symmetric inner function and R a root. Then the Borchers triples $(\mathcal{M}_R, U, \Omega)$ and $((\mathcal{M}_+ \otimes 1) \vee S_\varphi(1 \otimes \mathcal{M}_-)S_\varphi^*, U_+ \otimes U_-, \Omega_+ \otimes \Omega_-)$, obtained as deformations of the free massless chiral triple, are unitarily equivalent.

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- Other mass zero deformations also possible, related to scaling limits [Bostelmann/GL/Morsella 2011].
- Different choices of fields (different roots) lead to equivalent nets.
- Does there exist a unique Borchers triple for each "admissible" S-matrix?

Conclusions

- Borchers triples can be viewed as building blocks of QFTs
- A possible strategy for algebraic constructions of models proceeds via deformations of Borchers triples
- One example (warping) of such a deformation and a growing number of particular examples of Borchers triples known
- General deformation/structure theory wanted

Conclusions

- Borchers triples can be viewed as building blocks of QFTs
- A possible strategy for algebraic constructions of models proceeds via deformations of Borchers triples
- One example (warping) of such a deformation and a growing number of particular examples of Borchers triples known
- General deformation/structure theory wanted

» Borchers continues to inspire mathematical work in QFT, and lives on through his seminal contributions to the subject.