Wedge-Local Quantum Fields and Non-Commutative Minkowski Spacetime

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Local and Non-Local QFTs

- **Locality** is fundamental principle in QFT
- In recent developments, also non-local quantum fields are used:
  1. QFT on noncommutative spacetimes
  2. polarization-free generators (wedge-local quantum fields)
- **Here:** Discussion of a new model combining the two features

- **Plan of Talk:**
  1. Free field models on noncommutative Minkowski space
  2. A new model
  3. Wedges and wedge-local quantum fields
  4. Interaction and local observables
  5. Conclusion and open questions

1) Free fields on NC Minkowski space

- noncommutative coordinates: $[\hat{x}_\mu, \hat{x}_\nu] = iQ_{\mu\nu}$, $Q_{\mu\nu} = -Q_{\nu\mu}$
- motivated by classical gravity & uncertainty principle [DFR 95]
- usual choice of "noncommutativity parameter" Q:
  fixed, numerical matrix. Standard form

$$Q = \begin{pmatrix} 0 & \kappa_{\mathrm{e}} & 0 & 0 \\ -\kappa_{\mathrm{e}} & 0 & 0 & 0 \\ 0 & 0 & 0 & \kappa_{\mathrm{m}} \\ 0 & 0 & -\kappa_{\mathrm{m}} & 0 \end{pmatrix}, \qquad \kappa_{\mathrm{e}}, \kappa_{\mathrm{m}} \in \mathbb{R}\,.$$

- Problem: Fixed Q breaks Lorentz invariance

- ϕ: scalar free field on commutative Minkowski (dimension $d \geq 2$)
- noncommutative counterpart [Doplicher/Fredenhagen/Roberts 95]

$$\phi_{\otimes}(Q,x) := \int d\mu(p) \left(e^{ip\cdot x} a_{\otimes}^{*}(Q,p) + e^{-ip\cdot x} a_{\otimes}(Q,p) \right) ,$$

$$a_{\otimes}(Q,p)^{*} := e^{ip\cdot\hat{x}} \otimes a(p)^{*} , \qquad a_{\otimes}(Q,p) := e^{-ip\cdot\hat{x}} \otimes a(p) ,$$

- operator-valued distribution on $\mathcal{V} \otimes \mathcal{H}$,
  $\mathcal{V}$: Representation space of $\hat{x}$
  $\mathcal{H}$: Bose Fock space over $\mathcal{H}_1 := L^2(H_m^+, d\mu)$
- Commutation relations

$$a_{\otimes}(Q,p)a_{\otimes}(Q,p') = e^{-ipQp'} a_{\otimes}(Q,p')a_{\otimes}(Q,p) , \qquad pQp' := p_{\mu}Q^{\mu\nu}p'_{\nu} ,$$

$$a_{\otimes}(Q,p)a_{\otimes}^{*}(Q,p') = e^{+ipQp'} a_{\otimes}^{*}(Q,p')a_{\otimes}(Q,p) + \omega_{\boldsymbol{p}}\delta(\boldsymbol{p} - \boldsymbol{p}')\,\mathrm{id}_{\mathcal{V}\otimes\mathcal{H}} .$$

- $Q \neq 0 \Longrightarrow \phi_{\otimes}$ not local

[Fiore/Wess 07, Chaichian et. al. 04]:

- Consider n-point functions with Moyal-like product $\star_Q$

$$\langle\Omega\,,\phi(x_1)\star_Q\cdots\star_Q\phi(x_n)\Omega\rangle=\prod_{1\leq l<k\leq n}e^{-\frac{i}{2}\partial_l^\mu Q_{\mu\nu}\partial_k^\nu}\,\langle\Omega\,,\phi(x_1)\cdots\phi(x_n)\,\Omega\rangle_{\mathcal{H}}$$

- different strategy than in [DFR]
- usually combined with "twisted" Lorentz group

2) A new model

- Alternative representation of fields $\phi_\otimes(Q,x)$, on $\mathcal{H}$
[Akofor/Balachandran/Jo/Joseph 07, Grosse 79, ...]

$$a(Q,p) := e^{\frac{i}{2}pQP}a(p)\,, \qquad a^*(Q,p) := e^{-\frac{i}{2}pQP}a^*(p)\,,$$

with $P =$ (second quantized) generators of translations in "free field representation" U of Poincaré group on $\mathcal{H}$

- Corresponding field operators:

$$\phi(Q,x) := \int d\mu(p)\left(e^{ip\cdot x}\,a^*(Q,p) + e^{-ip\cdot x}\,a(Q,p)\right)\,.$$

- Same commutation relations as $\phi_\otimes(Q,x)$ for fixed Q
- Vacuum expectation values:

$$\begin{aligned}\langle\Omega,\,\phi(Q,x_1)\cdots\phi(Q,x_n)\Omega\rangle &= \langle\Omega,\,\phi(x_1)\star_Q\cdots\star_Q\phi(x_n)\Omega\rangle\\ &= \langle(\xi\otimes\Omega)\,,\phi_\otimes(x_1)\cdots\phi_\otimes(x_n)\,(\xi\otimes\Omega)\rangle\end{aligned}$$

Wightman properties of the fields $\phi(Q, x)$

- temp. distributions taking values in operators on Fock space $\mathcal{H}$,

$$f \longmapsto \phi(Q,f) = \int d^dx\, f(x)\phi(Q,x) = a^*(Q,f^+) + a(Q,f^-)\,,$$

$$f^\pm(p) := \int d^dx\, f(x)\, e^{\pm ip\cdot x}\,, \qquad p = (\omega_{\boldsymbol{p}}, \boldsymbol{p}) \in H_m^+\,.$$

- domain properties like Wightman fields, $\phi(Q,f)^* \supset \phi(Q,\overline{f})$
- Reeh-Schlieder property: For $\mathcal{O} \subset \mathbb{R}^d$ open,

$$\mathrm{span}\,\{\phi(Q,f_1)\cdots\phi(Q,f_n)\Omega \;:\; n \in \mathbb{N}_0,\, f_1, ..., f_n \in \mathscr{S}(\mathcal{O})\}$$

  is dense in $\mathcal{H}$.

- not local: $[\phi(Q,x), \phi(Q,y)]\Omega \neq 0$ for $(x-y)^2 < 0$ (if $Q \neq 0$)
- not covariant: $U(y,\Lambda)\phi(Q,x)U(y,\Lambda)^{-1} \neq \phi(Q,\Lambda x + y)$

Transformation properties

Adjoint action of U on $a^{\#}(Q,p)$ induces transformation of Q:

$$U(y,\Lambda)\phi(Q,x)U(y,\Lambda)^{-1} = \phi(\gamma_\Lambda(Q),\Lambda x + y)\,, \qquad (y,\Lambda) \in \mathcal{P}\,,$$

$$\text{with} \qquad \gamma_\Lambda(Q) := \begin{cases} \Lambda Q \Lambda^T & ; \quad \Lambda \in \mathcal{L}^\uparrow \\ -\Lambda Q \Lambda^T & ; \quad \Lambda \in \mathcal{L}^\downarrow \end{cases}\,, \qquad Q \in \mathbb{R}^-_{d\times d}\,.$$

- γ is an action of $\mathcal{L}$ on $\mathbb{R}^-_{d\times d}$.
- Role of Q similar to “Lorentz index”
- Fixed Q breaks Lorentz symmetry $\Rightarrow$ consider *family* of fields

$$\{\phi(Q,x)\ :\ Q \in \mathcal{Q}\}$$

with a γ-orbit $\mathcal{Q} \subset \mathbb{R}^-_{d\times d}$

- Commutation relations between $a^\#(Q,p)$ and $a^\#(Q',p')$ needed

Commutation relations for different Q:

$$
\begin{aligned}
a(Q,p)a(Q',p') &= e^{-\frac{i}{2}p(Q+Q')p'}a(Q',p')a(Q,p)\\
a^*(Q,p)a^*(Q',p') &= e^{-\frac{i}{2}p(Q+Q')p'}a^*(Q',p')a^*(Q,p)\\
a(Q,p)a^*(Q',p') &= e^{+\frac{i}{2}p(Q+Q')p'}a^*(Q',p')a(Q,p)\\
&\qquad\qquad + \omega_{\boldsymbol{p}}\delta(\boldsymbol{p}-\boldsymbol{p}')e^{\frac{i}{2}p(Q-Q')P}
\end{aligned}
$$

- Transformation behaviour $\phi(Q,x) \to \phi(\gamma_\Lambda(Q), \Lambda x + y)$ similar to string-localized fields of [Mund/Schroer/Yngvason 05]
- Interpretation of Q as localization region possible?
- Consider *relative* locality properties of the fields $\phi(Q,x)$

3) Wedges and wedge-local quantum fields

Idea:

Find set $\mathcal{W}_0$ of regions in $\mathbb{R}^d$, set $\mathcal{Q}$ of nc. parameters, and a map

$$Q : \mathcal{W}_0 \to \mathcal{Q}\,, \qquad Q(\Lambda W) = \gamma_\Lambda(Q(W))\,, \qquad W \in \mathcal{W}_0\,.$$

$\to$ need isomorphic homogeneous spaces $(\mathcal{W}_0, \iota) \cong (\mathcal{Q}, \gamma)$

Wedges in $\mathbb{R}^d$

- Reference region ("right wedge") $W_1 := \{x \in \mathbb{R}^d \,:\, x_1 > |x_0|\}$
- Sets of wedges:

$$\mathcal{W} := \mathcal{P} W_1 \qquad \text{(Poincaré transforms of } W_1)$$
$$\mathcal{W}_0 := \mathcal{L} W_1 \qquad \text{(Lorentz transforms of } W_1)$$

- $\mathcal{W}_0 =$ wedges with edges through origin,

$$\mathcal{W}_0 = \mathcal{L}_+^\uparrow W_1 \quad (d > 2), \qquad \mathcal{W}_0 = \{W_1, -W_1\} \quad (d = 2)\,.$$

  see also [Buchholz/Dreyer/Florig/Summers 98, Kuckert/Lorenzen 05]
- Reflections $j_\mu : x_\mu \mapsto -x_\mu$, $j := j_0 \cdots j_{d-1} : x \mapsto -x$
- $\hat{\mathcal{L}} := \mathcal{L}_+^\uparrow \cup j\mathcal{L}_+^\uparrow$ ($\hat{\mathcal{L}} = \mathcal{L}_+$ in even dimensions)
- $\mathcal{W}_0$ with $\hat{\mathcal{L}}$-action $\iota_\Lambda : W \mapsto \Lambda W$ is an $\hat{\mathcal{L}}$-homogenenous space

- Want $(\mathcal{W}_0, \iota) \cong (\mathcal{Q}, \gamma)$, with

$$\mathcal{Q} = \{\gamma_\Lambda(Q_1) \,:\, \Lambda \in \hat{\mathcal{L}}\,\}$$

  such that

$$Q : \mathcal{W}_0 \to \mathcal{Q}\,, \qquad Q(\Lambda W_1) := \gamma_\Lambda(Q_1)$$

  is well-defined homomorphism of $\hat{\mathcal{L}}$-homogeneous spaces.
- Stabilizer groups $\hat{\mathcal{L}}(W_1, \iota)$ and $\hat{\mathcal{L}}(Q_1, \gamma)$ must coincide $\to$ fix Q_1

Lemma

$Q(\Lambda W_1) := \gamma_\Lambda(Q_1)$ is a well-defined isomorphism of $\hat{\mathcal{L}}$-homogeneous spaces iff

$$Q_1 = \begin{pmatrix} 0 & \kappa_{\rm e} & 0 & \cdots & 0 \\ -\kappa_{\rm e} & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 0 \end{pmatrix} (d \neq 4), \quad Q_1 = \begin{pmatrix} 0 & \kappa_{\rm e} & 0 & 0 \\ -\kappa_{\rm e} & 0 & 0 & 0 \\ 0 & 0 & 0 & \kappa_{\rm m} \\ 0 & 0 & -\kappa_{\rm m} & 0 \end{pmatrix} (d = 4),$$

and $Q_1 \neq 0$. Then

- $\mathcal{Q} = \{\gamma_\Lambda(Q_1) \,:\, \Lambda \in \mathcal{L}_+^\uparrow\} \qquad (d > 2)$
- $\mathcal{Q} = \{Q_1, -Q_1\} \qquad (d = 2)$

and

- $Q(W') = -Q(W), \qquad W \in \mathcal{W}_0\,.$

Proof:

- Well-definedness: Compute stabilizer groups.
- Compatability with causal complement:

  Note $W_1' = -W_1$.

  $\Longrightarrow W' = -W = jW, \quad W \in \mathcal{W}_0 = \mathcal{L}W_1$.

  $\Longrightarrow Q(W') = Q(jW) = \gamma_j(Q(W)) = -jQ(W)j^T = -Q(W)$.

Wedge-local quantum fields

- With the isomorphism $Q : \mathcal{W}_0 \to \mathcal{Q}$, define, $W \in \mathcal{W}_0$,

$$\begin{aligned}\phi_W(x) &:= \phi(Q(W), x) \\ &= \int d\mu(p) \left(a^*(Q(W), p)\, e^{ip\cdot x} + a(Q(W), p)\, e^{-ip\cdot x} \right)\end{aligned}$$

- Direct consequence of construction of Q:

$$U(y, \Lambda)\phi_W(x)U(y, \Lambda)^{-1} = \phi(\gamma_\Lambda(Q(W)), \Lambda x + y) = \phi_{\Lambda W}(\Lambda x + y)$$

- $\phi_W(x)$ describes an extended field configuration in $W + x$
- Is $\phi_W(x)$ *localized* in $W + x$ in the sense that

$$[\phi_W(x), \phi_{\tilde{W}}(y)] = 0\,, \qquad (W + x) \subset (\tilde{W} + y)' \quad \textbf{?}$$

Causal configurations of wedges and fields

- Let $W, \tilde{W} \in \mathcal{W}_0$.

$$W \subset \tilde{W}' \Longrightarrow \tilde{W} = W' + a$$

Sufficient to consider wedge W_1, i.e. the field commutator

$$[\phi_{W_1}(f), \phi_{W_1'}(g)]\,, \qquad f \in C_0^\infty(W_1),\; g \in C_0^\infty(W_1')$$

- Wedge-locality is an essentially two-dimensional problem
- Recall algebra of the $a^\#(Q,p)$ and $Q(W') = -Q(W)$.

$$\begin{aligned}
[a(Q(W),p),\, a(Q(W'),p')] &= 0 \\
[a^*(Q(W),p),\, a^*(Q(W'),p')] &= 0 \\
[a(Q(W),p),\, a^*(Q(W'),p')] &= \omega_{\boldsymbol{p}}\delta(\boldsymbol{p}-\boldsymbol{p}')e^{ipQ(W)P}
\end{aligned}$$

- $[\phi_{W_1}(f), \phi_{-W_1}(g)] = [a(Q_1, f^-), a^*(-Q_1, g^+)] + [a^*(Q_1, f^+), a(-Q_1, g^-)]$
- Evaluate field commutator on $\Psi \in \mathcal{D}$:

$$([\phi_{W_1}(f), \phi_{-W_1}(g)]\Psi)_n(q_1, ..., q_n)$$
$$= \int d\mu(p) \left(e^{ipQ_1 q} f^-(p) g^+(p) - e^{-ipQ_1 q} f^+(p) g^-(p)\right) \cdot \Psi_n(q_1, ..., q_n)$$

- perform analytic continuation of integrand in $\vartheta := \sinh^{-1} \frac{p_1}{(m^2+p_2^2+...+p_{d-1}^2)^{1/2}}$ from $\mathbb{R}$ to $\mathbb{R} + i\pi$:

$$f^- \to f^+, \quad g^+ \to g^-, \quad e^{ipQ_1 q} \to e^{-ipQ_1 q}$$

- **Result:** $[\phi_{W_1}(f), \phi_{-W_1}(g)]\Psi = 0$ for $\operatorname{supp} f \subset W_1$, $\operatorname{supp} g \subset -W_1$ and $\kappa_{\mathrm{e}} \geq 0$.

Proposition

ϕ_W is a temperate quantum field with the following properties, $W \in \mathcal{W}_0$

- Covariance: ($y, x \in \mathbb{R}^d$, $\Lambda \in \hat{\mathcal{L}}$, $W \in \mathcal{W}_0$)

$$U(y,\Lambda)\phi_W(x)U(y,\Lambda)^{-1} = \phi_{\Lambda W}(\Lambda x + y)\,.$$

- Wedge-Locality: ($x, y \in \mathbb{R}^d$, $W, \tilde{W} \in \mathcal{W}_0$)

$$(W + x) \subset (\tilde{W} + y)' \Longrightarrow [\phi_W(x), \phi_{\tilde{W}}(y)] = 0\,.$$

- Reeh-Schlieder property ($\mathcal{O} \subset \mathbb{R}^d$ open):

$$\overline{\text{span}\{\phi_W(f_1)\cdots\phi_W(f_n)\Omega \;:\; n \in \mathbb{N}_0,\ f_1,\dots,f_n \in \mathscr{S}(\mathcal{O})\}} = \mathcal{H}\,.$$

- Interpretation on "commutative" Minkowski space possible.

4) Interaction and local observables

The two-dimensional case, relation to integrable models

- In $d = 2$, $\mathcal{W}_0 = \{W_1, -W_1\}$ and $\mathcal{Q} = \{Q_1, -Q_1\}$.
  Isomorphism $Q : \mathcal{W}_0 \to \mathcal{Q}$ is

$$Q(\pm W_1) = \pm Q_1 = \pm\kappa_{\mathrm{e}} \begin{pmatrix} 0 & +1 \\ -1 & 0 \end{pmatrix} .$$

- Put $p(\vartheta) := m(\cosh\vartheta, \sinh\vartheta) \in H_m^+$. Then

$$e^{ip(\vartheta_1)Q_1 p(\vartheta_2)} = e^{i\kappa_{\mathrm{e}} m^2 \sinh(\vartheta_1 - \vartheta_2)} =: S_2(\vartheta_1 - \vartheta_2)$$

- S_2 is a *scattering function* for $\kappa_{\mathrm{e}} \geq 0$, i.e. bounded & analytic on $\{\zeta \, : \, 0 < \operatorname{Im}\zeta < \pi\}$ and

$$\overline{S_2(\vartheta)} = S_2(\vartheta)^{-1} = S_2(\vartheta + i\pi) = S_2(-\vartheta) .$$

- Let

$$z(\vartheta) := a(-Q_1, p(\vartheta)) , \qquad z(\vartheta)' := a(Q_1, p(\vartheta)) .$$

- Commutation relations:

$$z(\vartheta_1)z(\vartheta_2) = S_2(\vartheta_1-\vartheta_2)\,z(\vartheta_2)z(\vartheta_1)$$
$$z(\vartheta_1)z^\dagger(\vartheta_2) = S_2(\vartheta_2-\vartheta_1)\,z^\dagger(\vartheta_2)z(\vartheta_1) + \delta(\vartheta_1-\vartheta_2)\cdot 1$$

$\rightarrow$ representation of Zamolodchikov-Faddeev alg. with sc. fctn S_2

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$$z(\vartheta_1)'z(\vartheta_2)' = S_2(\vartheta_1-\vartheta_2)^{-1}\,z(\vartheta_2)'z(\vartheta_1)'$$
$$z(\vartheta_1)'z^\dagger(\vartheta_2)' = S_2(\vartheta_2-\vartheta_1)^{-1}\,z^\dagger(\vartheta_2)'z(\vartheta_1)' + \delta(\vartheta_1-\vartheta_2)\cdot 1$$

$\rightarrow$ representation of Zamolodchikov-Faddeev alg. with sc. fctn S_2^{-1}

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$$z(\vartheta_1)z(\vartheta_2)' = z(\vartheta_2)'z(\vartheta_1)\,,$$
$$z(\vartheta_1)z^\dagger(\vartheta_2)' = z^\dagger(\vartheta_2)'z(\vartheta_1) + \delta(\vartheta_1-\vartheta_2)\cdot e^{-ip(\vartheta_1)Q_1P}\,.$$

$\rightarrow$ “compatibility relations“ between $z^\#(\vartheta)$ and $z^\#(\vartheta)'$, implying relative wedge-locality of $\phi(x)=\phi_{W_1}(x)$ and $\phi'(x)=\phi_{-W_1}(x)$.

- In $d = 2$, the algebraic structure coincides precisely with the structure of an integrable qft with scattering function

$$S_2(\vartheta) = e^{im^2\kappa_{\rm e}\sinh\vartheta}$$

[Schroer 97, GL 03, Buchholz/GL 04, GL 06]

- But S_2 is not *regular* in the sense that it is bounded and analytic on $\{-\beta < \operatorname{Im}\vartheta < \pi + \beta\}$ with $\beta > 0$
- Known existence/structure theorems for local observables [Buchholz/GL 04, GL 06] do not apply here
- Status of local observables presently unclear
- Non-locality expected from relation to noncommutative spacetime

Recall concept of a polarization-free generator (PFG) [Schroer 98, Borchers/Buchholz/Schroer 00]:

- PFG = Wedge-localized operator generating single particle states from the vacuum (”no vacuum polarization“)
- PFGs with behaviour w.r.t. translations can be used for the calculation of two-particle scattering states *(temperate PFGs)*
- In spacetime dimension $d \geq 3$, temperate PFGs exist only in interaction-free theories

In the present model:

- $\phi_W(f)$ is a temperate PFG localized in wedge $(W + \mathrm{supp}\, f)''$
- Typical two-particle collision states:

$$(f^+ \times g^+)_{\mathrm{out}}(p_1, p_2) = e^{\frac{i}{2}p_1 Q_1 p_2} f^+(p_1) g^+(p_2) + e^{-\frac{i}{2}p_1 Q_1 p_2} f^+(p_2) g^+(p_1)$$
$$(f^+ \times g^+)_{\mathrm{in}}(p_1, p_2) = e^{-\frac{i}{2}p_1 Q_1 p_2} f^+(p_1) g^+(p_2) + e^{\frac{i}{2}p_1 Q_1 p_2} f^+(p_2) g^+(p_1)$$

- Non-trivial S-matrix for $Q_1 \neq 0$, similar to scattering in integrable models
- Wedge-local, covariant, interacting QFT in any dimension

Temperate PFGs and interaction in $d \geq 3 \Longrightarrow$ Contradiction with [BBS] ?

- In [BBS], local theory was assumed
- Here: Only wedge-local theory, no a priori control over local observable content
- $\Longrightarrow$ Restrictions on local observable content for $d \geq 3$

Algebraic formulation of the model:

- Wedge algebras

$$\mathcal{A}(W+x) := \{e^{i\phi_W(f)} \,:\, f \in \mathscr{S}_{\mathbb{R}}(W+x)\}'' \,, \qquad W \in \mathcal{W}_0$$

- Defines wedge-local, $\hat{\mathcal{P}}$-covariant net $\mathcal{W} \ni W \longmapsto \mathcal{A}(W) \subset \mathcal{B}(\mathcal{H})$
  - Ω cyclic and separating for each $\mathcal{A}(W)$, $W \in \mathcal{W}$
  - Wedge-duality: $\mathcal{A}(W') = \mathcal{A}(W)'$
- Algebras of local observables:

$$\mathcal{A}(\bigcap_k W_k) := \bigcap_k \mathcal{A}(W_k)$$

$\{W_k\} \subset \mathcal{W}$ family of wedges ($\bigcap_k W_k$ can be bounded)

Non-Local Properties of the Model

Assumptions of [BBS]:

- Wedge algebras locally generated ($\mathscr{O}$:= set of double cones),

$$\mathcal{A}(W) = \bigvee_{\mathscr{O} \ni \mathcal{O} \subset W} \mathcal{A}(\mathcal{O})$$

  $\longrightarrow$ not valid here

- Local Reeh-Schlieder property:

$$\overline{\mathcal{A}(\mathcal{O})\Omega} = \mathcal{H}\,, \qquad \mathcal{O} \in \mathscr{O}$$

  $\longrightarrow$ not valid here

- Similar situation found in [Buchholz/Summers 06]
- Complete absence of local observables, i.e. $\mathcal{A}(\mathcal{O}) = \mathbb{C} \cdot 1$? Not decided yet.

- Model defined by the fields ϕ_W is not generated by a local QFT

5) Conclusions and open questions

New model QFT with two different interpretations:

1. Theory containing an infinity of free field models on noncommutative Minkowski spaces with different noncommutativity parameters $Q \in \mathcal{Q}$
   - Consequences of application of proper, untwisted Poincaré group as symmetry group in Fock space representation explored
   - Relative localization of theories with fixed nc. parameter found
   - Formulation of new models on NC spaces? $\int dQ\, \phi(Q, x)$ ...
2. Non-local, but wedge-local QFT on “commutative” Minkowski space
   - First example of a wedge-local, fully covariant, interacting QFT in higher dimensions
   - Temperateness of PFGs / simple form of the S-matrix
   - $\Longrightarrow$ Non-local behaviour / Restrictions on local observable content
   - Formulation of models with “controlled” non-locality?